The QCD CP - Problem

or, the road to the axion (in my head)

① the "$U(1)_A$ problem"

② the QCD vacuum

③ the "strong-CP problem"!

(good notes: Jacob Schwichtenburg axion pages)

④ the solutions...

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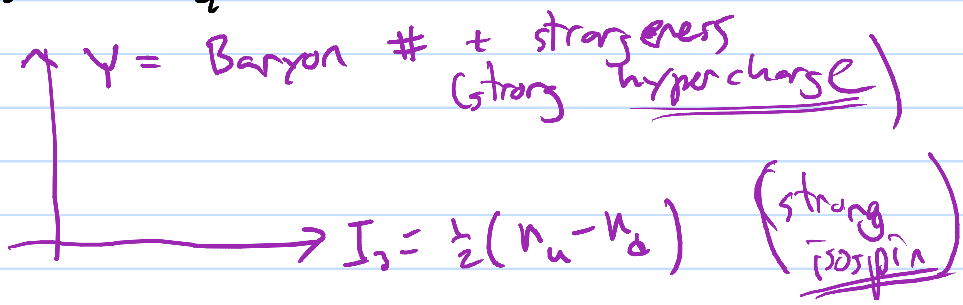
1) strong isospin (with historical-ish eyes)

- firstly, don't confuse this with weak isospin
  - weak isospin is more like charge under the weak SU(2) gauge group... eg, the object $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$, which is in the doublet (fundamental) representation of SU(2): ν_e has weak-isospin $+\frac{1}{2}$
- in contrast, strong isospin is a quantum number that hadrons have

- eg, the Delta baryon 4-plet:

$$\underset{ddd}{\Delta^-} \quad \underset{ddu}{\Delta^0} \quad \underset{duu}{\Delta^+} \quad \underset{uuu}{\Delta^{++}}$$

- historically, they saw these particles were exchangeable with respect to the strong force
  - if you go to higher energy, they found two conserved quantities:



now, the real underlying symmetry is more obvious... quark flavors

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2) Flavour symmetry (with modern eyes)

- u, d (, s) exchange symmetry
- must have same masses
  - massless ⇒ chiral theory - no distinguishing L/R particles
  - of course, quarks have masses... but for these 3, $m_q < \Lambda_{QCD}$

- we have then an <u>approximate</u> symmetry.

$\Rightarrow$ we can still have noether currents corresponding to the symmetry, but they have small corrections $\propto \frac{m_q}{\Lambda_{QCD}}$

- In detail, the symmetry is

$$\begin{pmatrix} u \\ d \\ \vdots \end{pmatrix} \to U \begin{pmatrix} u \\ d \\ \vdots \end{pmatrix}, \quad U \in \mathrm{rep}(U(N))_{\text{fund.}}$$

- N = # quarks participating (usually 2/3, we'll stick to 2 mainly for ease)

- $U(N) = SU(N) \times U(1)$

$$\downarrow$$

$$SU(N)_L \times SU(N)_R \times U(1)_L \times U(1)_R$$

- is a <u>global</u> symmetry of the Standard Model Lagrangian

- An $SU(2)$ symmetry acts like:

$$\psi \equiv \begin{pmatrix} u \\ d \end{pmatrix} \to \psi' = e^{-i\underset{\sim}{\tau}\cdot\underset{\sim}{\theta}}\,\psi$$

Pauli matrices

- One of particle physicists favorite tricks is to change the basis which defines particles...

- we swap L/R for vector / axial:

$\psi_V = (\psi_L + \psi_R)/2, \qquad \psi_A = (\psi_L + \psi_R)/2$

$\downarrow$ transforms like vector under Parity

$\downarrow$ ... axially ... (eg, cross-product)

- In this basis, the symmetry is

$$SU(N)_V \times U(1)_V \times SU(N)_A \times U(1)_A$$

3) the $U(1)_A$ problem

- the most important question: this is a symmetry of $\mathcal{L}$, but do we actually see it??

- $SU(2)_V \times U(1)_V \longleftrightarrow SU(2)_I \times U(1)_B$

$SU(2)_I \downarrow$ strong isospin!

$U(1)_B \downarrow$ baryon #!

- what about the axial bit?

consider a quark condensate $\langle 0 | \bar{Q} Q | 0 \rangle \neq 0$

(which exist, kinda like Cooper pairs, popping out of the vacuum ... idk!)

classic exercise to rewrite as

$$\langle 0 | \bar{Q} Q | 0 \rangle = \langle 0 | \overline{Q_L} Q_R + \overline{Q_R} Q_L | 0 \rangle$$

if $Q_L \to U_L Q_L$, $Q_R \to U_R Q_R$, this is

only invariant if $U_L = U_R$ ... $\Leftrightarrow$ vector part

So... there is no $SU(2)_A \times U(1)_A$ symmetry?

$\Rightarrow$ it must be spontaneously broken

(ie, the vacuum state is not invariant under the symmetry degrees of freedom $\rightsquigarrow$ goldstone bosons)

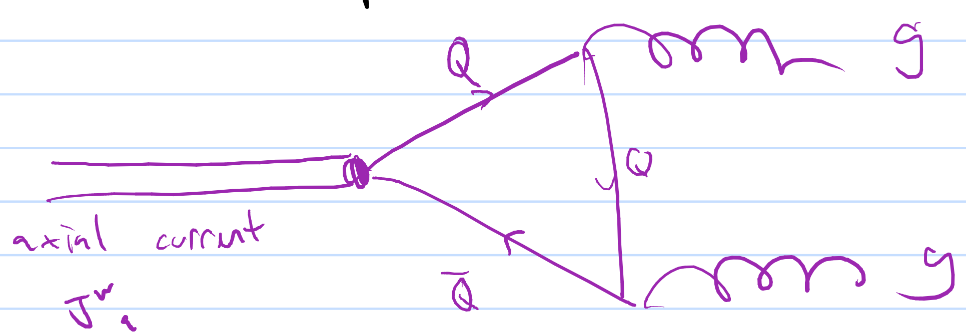
So... are there 3+1 pseudoscalar mesons to be these? (8+1 if 3 quarks)?

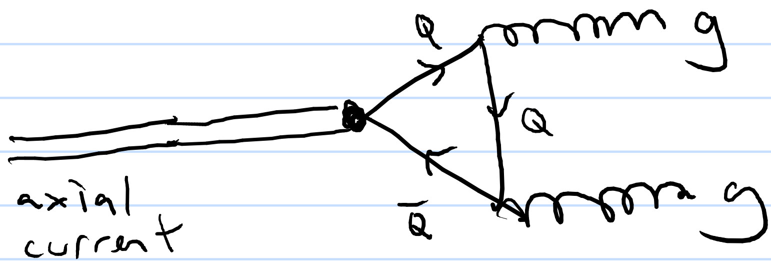
- 3 pions are natural candidates... masses are light, as required by the approximate symmetry
- the fourth, η-meson, is not light though, so we can only account for $SU(2)_A$

So... there is no $U(1)_A$ symmetry?

$\Rightarrow$ must be explicitly broken, at the quantum level. "anomaly"

For example...





$J^{\mu}_{a} = \bar{\psi}\gamma^{\mu}\gamma_5\psi$

can Find $\partial_\mu J^\mu_A \propto G_{\mu\nu}\tilde{G}^{\mu\nu a}$

- Ok, so... what does this mean?
  - IF I apply the $U(1)_A$ transformation to the S.M. $\mathcal{L}$, it must transform like:

$$\mathcal{L} \to \mathcal{L} + \frac{\alpha}{16\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu a}$$

- but... $G\tilde{G}$ can be written like $\partial_\mu k^\mu$, which means its a total derivative

$\Rightarrow$ if $A^{\mu a} = 0$ at infinity, this vanishes. Meaning the $U(1)_A$ symmetry, which is not conserved in experiment, is conserved in $\mathcal{L}$!

- $\boxed{\text{this is the } U(1)_A \text{ problem}}$

4) Pure gauge states & the vacuum

- the answer to the $U(1)_A$ problem is the assumption that $A^{vac} = 0$
  - Under $U \in$ gauge group, A transforms

$$A_\mu \to U A_\mu U^\dagger - \frac{i}{g} U \partial_\mu U^\dagger$$

$$A_\mu = 0 \Rightarrow A_\mu^{pg} = -\frac{i}{g} U \partial_\mu U$$

A_μ^{pg} are called <u>pure-gauge</u> states

- Let's investigate these vacua, which are defined by U

assumptions: 1) $A_0 = 0$ (temporal gauge)

2) $U(x) = 1$ when $|x| \to \infty$

$A_\mu = 0$ satisfies this... we will see why this is not so ad-hoc in a bit.

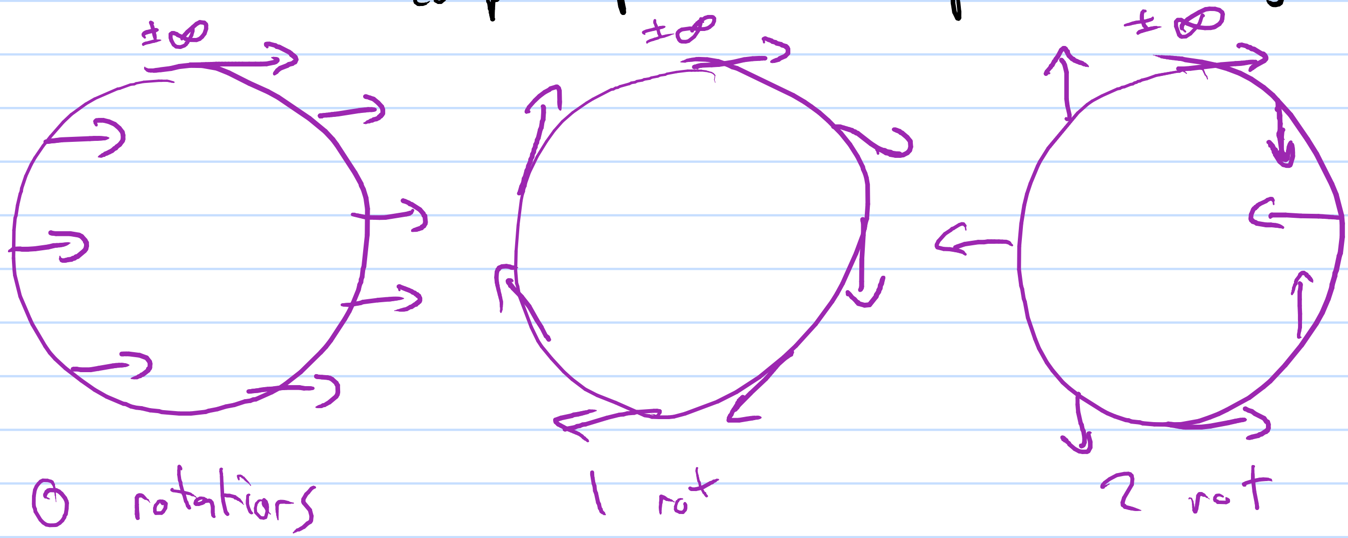
3) $u(x)$ continuous (else A_μ is broken!)

- Let's go slow and look first at a $U(1)$ gauge group in $1+1$ spacetime

  - assumption $2 \Rightarrow x \in \underline{\underline{S}}^1$, a circle, since $-\infty$ and $+\infty$ are identified and $\mathbb{R}$ has been compactified

  - $A_\mu^{PG} \propto U \partial_\mu U$ is defined by U, so how $U(x)$ is defined on S^1 defines A_μ^{PG}

  - Let's draw S^1 and then use arrows in the complex plane to represent $U(x)$



- can intuit without any topology knowledge that no cts transformations U applied to these will swap one into another without "breaking" somewhere

- each PG-state in indexed by a "winding number" $\in \mathbb{Z}$

imagine, wrapping a rubber band around a cylinder. you can stretch it and twist it around again some integer number of times

- in topology-world, $\pi_1(S^1) \cong \mathbb{Z}$

- For us, we really have $SU(3)$ and $\mathbb{R}^3$
  - $SU(2)$ is a subgp of $SU(3)$, so its fine to just use it (apparently!)
  - $\mathbb{R}^3$ is compactified to S^3, a hard-to-imagine 3-sphere
  - the topological question is now, how can we index $SU(2)$ wrapped around S^3?

Luckily, $S^3 \cong SU(2)$!

and, unsurprisingly,

$\pi_3(S^3) \cong \mathbb{Z}$!

- All up: there are distinct vacua in our theory, indexed by an integer. But what happens when $U(x) \neq 1$ at infinity?

For the sake of not getting technical, lets say $A^0 \to A^1$ transitions like

$A_\mu^\beta = \beta A_\mu^1$ for some β parameter (could be $\beta(x)$, who cares)

For $0 < \beta < 1$, lets look at the field-strength $G_{\mu\nu}$

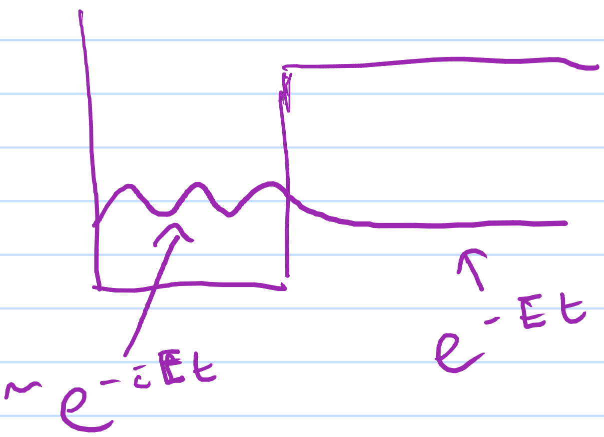
$$
\begin{aligned}
G_{\mu\nu} &\equiv \partial_\mu A_\nu^\beta - \partial_\nu A_\mu^\beta + [A_\mu^\beta, A_\nu^\beta] \\
&= \beta(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1) + \beta^2 [A_\mu^1, A_\nu^1] \\
&= (\beta^2 - \beta)[A_\mu^1, A_\nu^1] \quad \text{using } A^1 \text{ is PG so } G_{\mu\nu} = 0 \\
&\neq 0 \; !
\end{aligned}
$$

$$\text{Energy} \propto \int G^a_{\mu\nu} G_{a\mu\nu}\, d^3x \neq 0 \; !$$

- Passing from one distinct PG vacuum to another means passing through a <u>potential energy barrier</u>

5) tunnelling & instantons

- we are looking then for a finite-energy field configuration, that allows us to tunnel between vacua.
  - It just so happens, there is a nifty way to find such solutions...
  - consider the usual QM potential well:



- we see that the tunneling part is the usual solution, transformed $t \to it$
- if we transform our field equations from Minkowski to Euclidean space (ie $t \to it$), we can solve them to find tunelling solutions called "instantons"

- in this case we want a solution which changes winding # by 1 (higher ones are supressed relative to this)

  - such a solution is known: the "BPST" instanton (the form is boring, you can look it up)

- when you tunnel, you can pick up a phase which is just an invariant parameter in the theory
  - I think of this as ~ the width but its hard to get good intuition!

- so we can define the true QCD vacuum as a sum of all the vacua, with some phase between them:

$$\boxed{|\theta\rangle \equiv \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle}$$

6) The QCD CP-Problem (almost there!)

- What is the effect of the complicated QCD vacuum,

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle \quad ?$$

- In our path integral, we sum over all paths. Some of those paths include moving between vacua, which is a finite-energy process. We can recalculate the vacuum-vacuum transition $\langle\theta|e^{-H\tau}|\theta\rangle$

- With apologies again for the lack of easy physical intuition, we find that accounting for this adds a term to the Lagrangian:

$$\Delta\mathcal{L} = \frac{\theta}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

... which is proportional to the axial anomaly term: $\frac{\alpha}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$!

(whenever there is such a coincidence, I assume I'm missing something fundamental...)

- What is really going on is that $u(1)_A$ transformations are not "disappearing" terms: really they are changing the vacuum by $\theta \to \theta + \alpha$
  - then the $\underline{u(1)_A \text{ problem is solved}}$... its not invariant in real life, or the Lagrangian!

- We're not quite done, though. Because quark masses mix under the weak force, (which $\underline{\text{is}}$ chiral), if we want to fix them we must use the α axial transformation freedom: $\psi_L \to e^{-i \operatorname{Arg} \operatorname{Det}(m)} \psi_L$, $\psi_R \to e^{+i\cdots} \psi_R$

  - so $\bar{\theta} \equiv \theta + \operatorname{Arg} \operatorname{Det}(m)$ is totally fixed by the two unrelated weak and strong forces.

- $\bar{\Theta}$ appears as a physical parameter in calculations such as the nuetron electric dipole moment
  - It shows up in CP violating strong interactions
  - I find these calculations less fun and more hard...
- turns out... $\bar{\Theta} < 10^{-9}$

why is it so small??

how do the weak + strong bits cancel each other out so nicely??

$\boxed{\text{this is the QCD CP problem.}}$

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