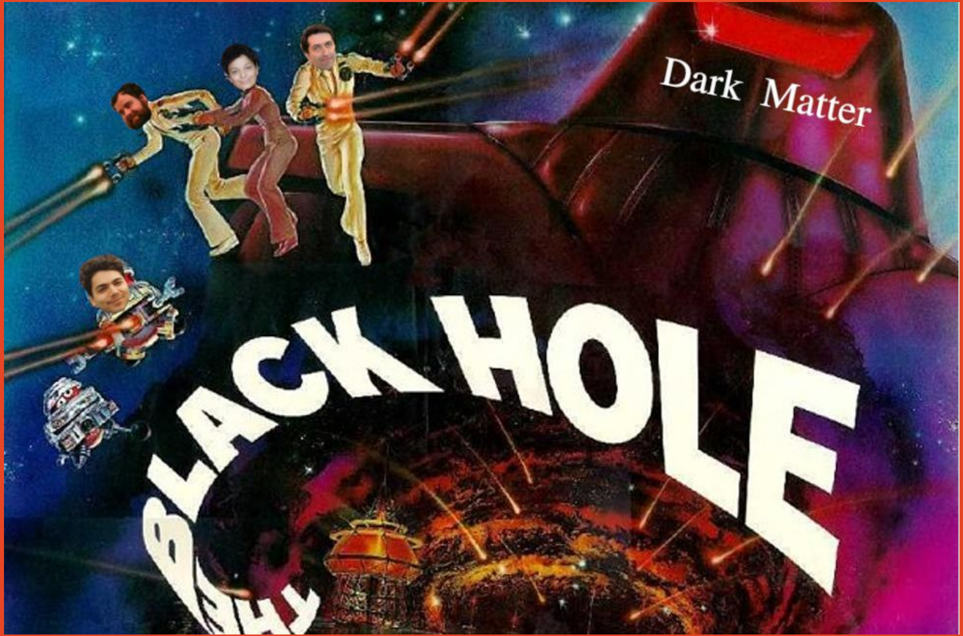
Cosmological PBHs as Dark Matter

Zachary S.C. Picker

Sydney CPPC seminar



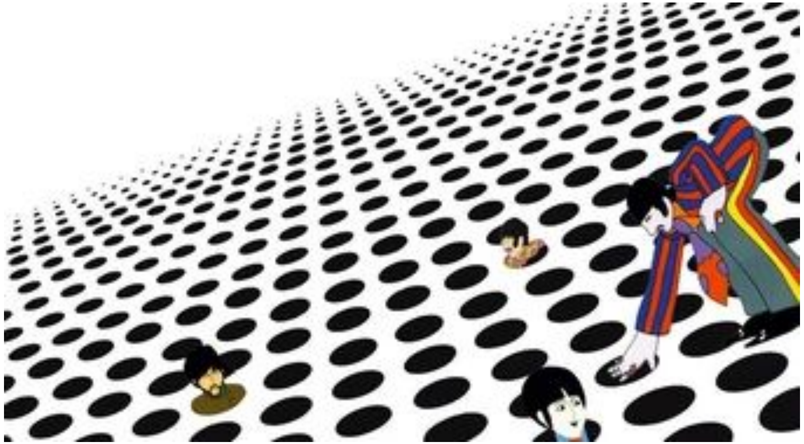
THE UNIVERSITY OF SYDNEY

June 2021

Zachary S. C. Picker,
University of Sydney

Outline

1. PBH dark matter
   a. Physics
   b. Constraints
2. Cosmological PBH solutions
   a. General considerations
   b. Thakurta PBHs
3. Phenomenology:
   a. LIGO binary abundance bounds
   b. Evaporation bounds
4. Conclusions



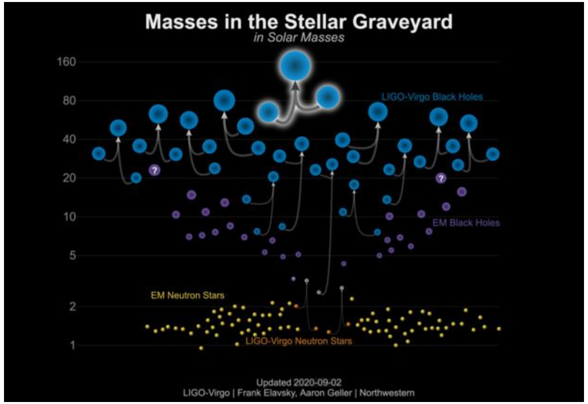
1. PBH Dark Matter

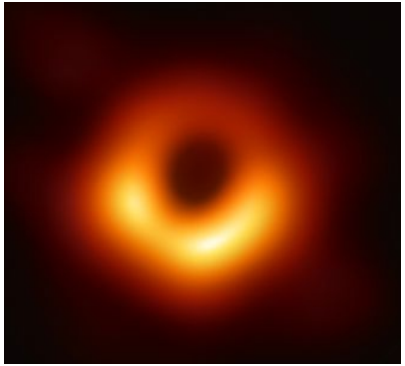
Zachary S. C. Picker,
University of Sydney

Motivation

- Only observed DM candidate...
- May be hard to explain BH observations from stellar collapse
  - Pair-instability mass gap: ~50-150 M_{sun}
  - How do we seed supermassive BHs?

⇒ Primordial?





EHT Collaboration, 2019
LIGO-Virgo/ Northwestern U. / Frank Elavsky & Aaron Geller, 2020

Carr et al, 2020

Zachary S. C. Picker,
University of Sydney

Formation

- Primordial black holes form from horizon-size overdensities:

$$m = \gamma m_H$$

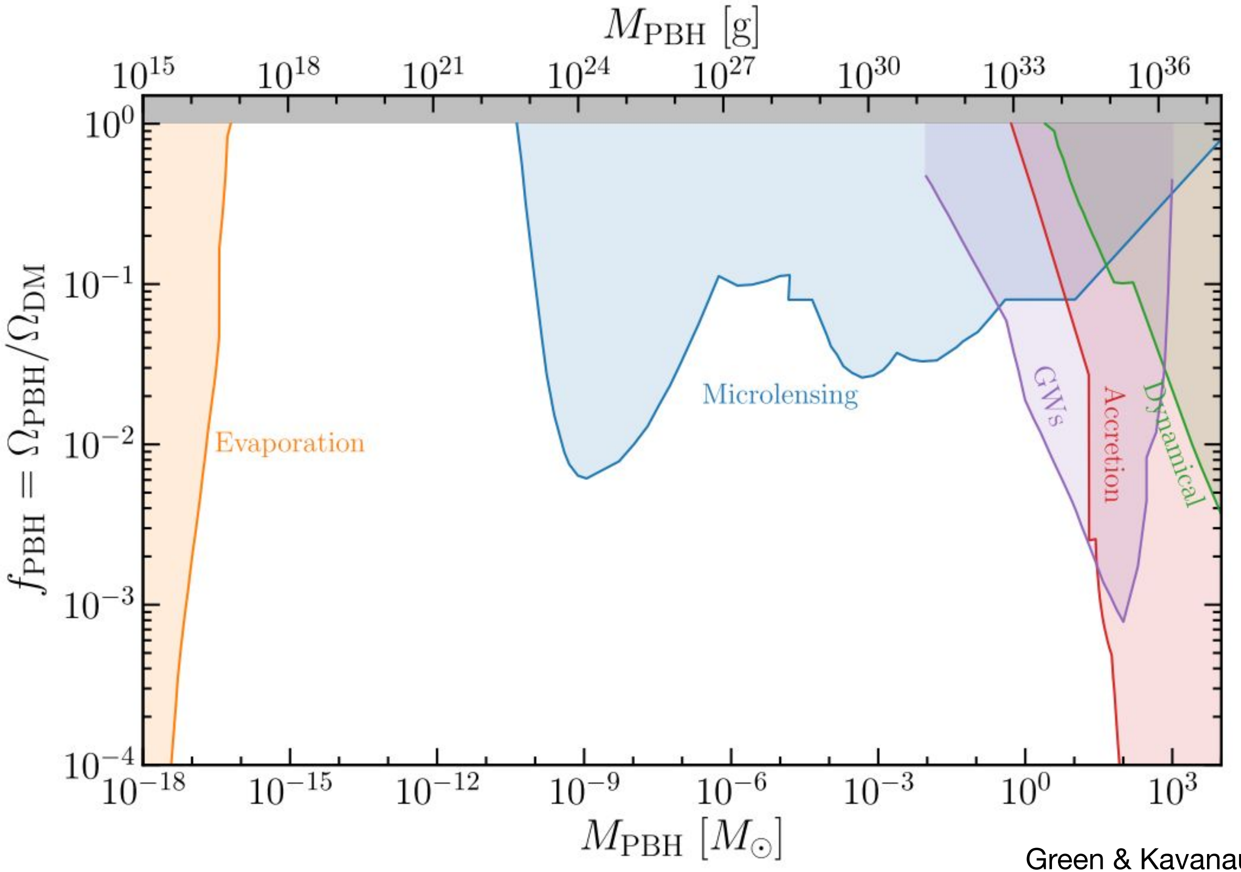
$$\approx 2.03 \times 10^5 \, \gamma \left(\frac{t}{1\,\mathrm{s}} \right) M_\odot$$

$$\gamma \approx 0.2$$

- Inhomogeneities in early universe is common mechanism
  - Also: cosmic loop collapse, domain walls, other exotic things
- Often assume ~monochromatic mass spectrum

Zachary S. C. Picker,
University of Sydney

Constraints



Particularly care about:

Which constraints rely on early-universe calculations?

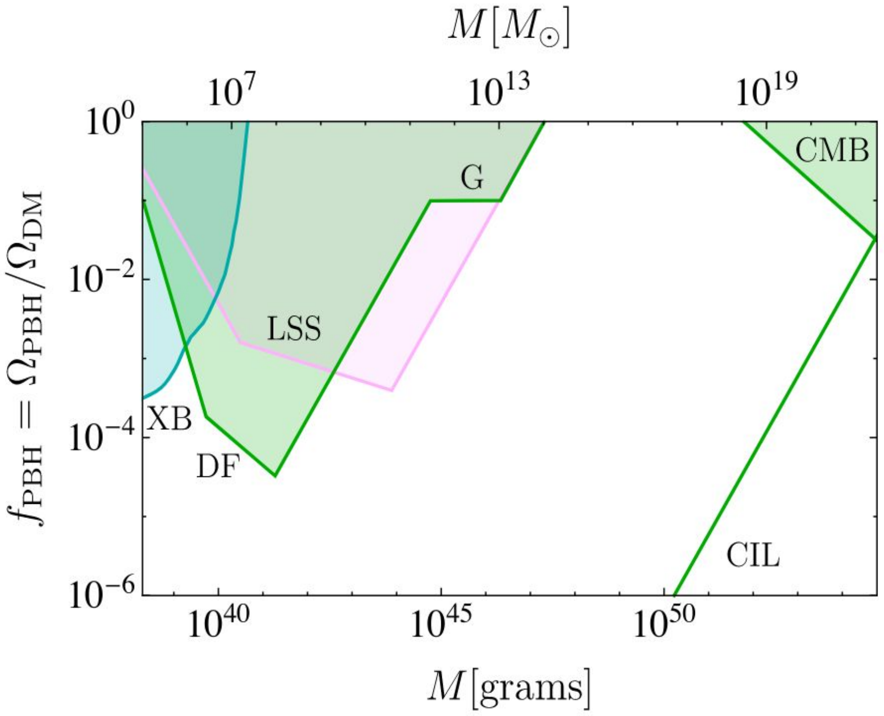
Green & Kavanaugh, 2020

Zachary S. C. Picker,
University of Sydney

"Stupendously Large" PBH Constraints

$M > 10^5 M_{sun}$

- DF: halo dynamical friction
- G: galaxy tidal distortions
- CMB: dipole
- LSS: large scale structure formation
- XB: X-ray binary accretion
- CIL: cosmological incredulity



Carr et al, 2020

Zachary S. C. Picker,
University of Sydney

"LIGO Range" bounds

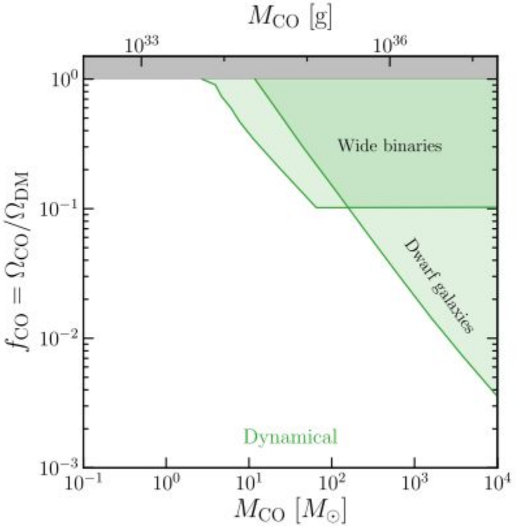
~1M_{sun} < M < ~1000 M_{sun}

Dynamical:

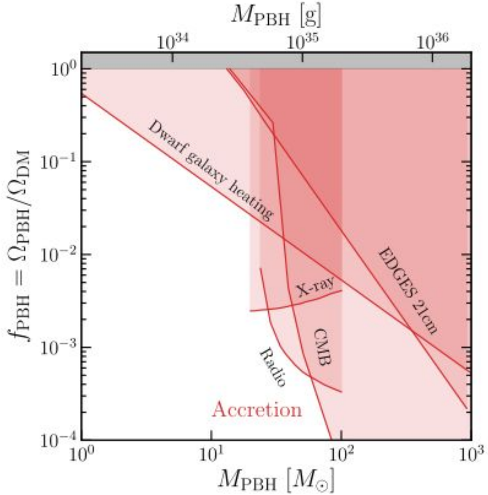
- Wide binaries
- Dwarf galaxies/star clusters (Eridanus II)

Accretion:

- Recombination
- CMB
- X-ray/radio



Green & Kavanaugh, 2020



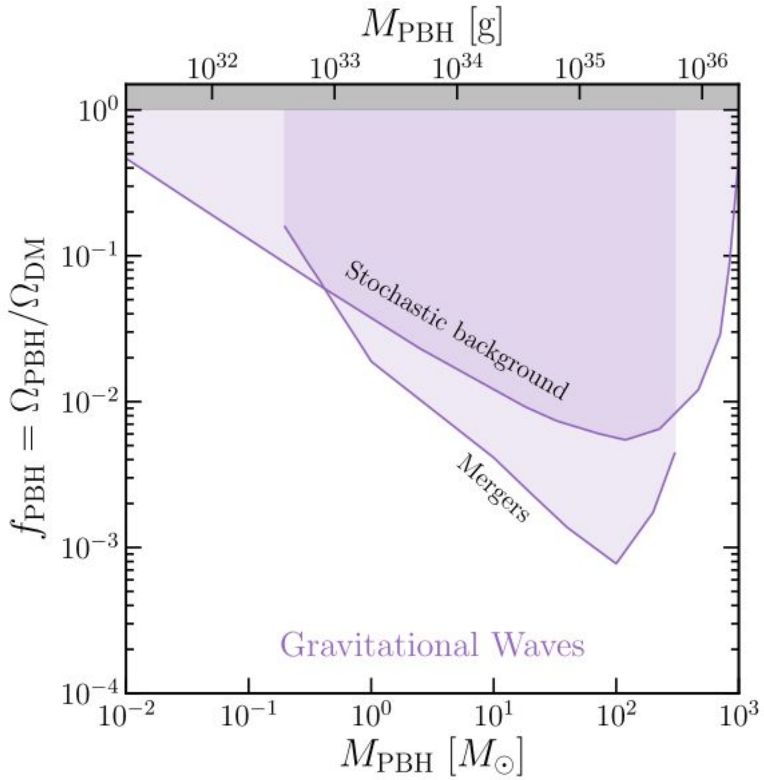
Zachary S. C. Picker,
University of Sydney

"LIGO Range" bounds

$\sim 1 M_{sun} < M < \sim 1000 M_{sun}$

Gravitational waves

- Binary abundance
  - Mergers today
  - Stochastic background

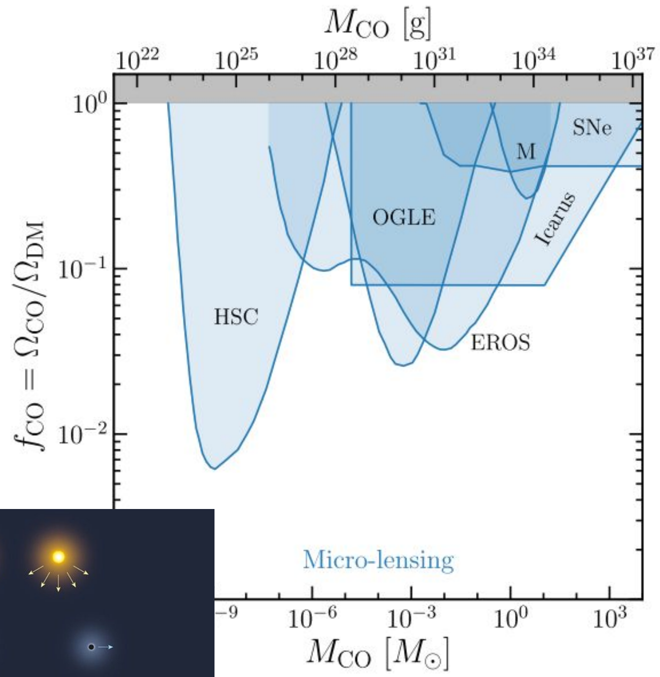


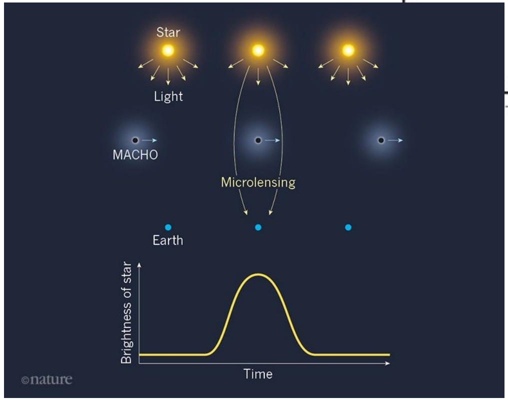
Green & Kavanaugh, 2020

Microlensing- range constraints

$\sim 10^{-12}\, M_{sun} < M < \sim 10\, M_{sun}$

- MACHO,EROS,OGLE,HSC:
  - stellar microlensing
  - LMC, galactic bulge, M31
- Icarus: lensing event
  - Would be dimmer with compact object pop.
- SNe: supernova magnification distribution
  - Depends on DM smoothness





Green & Kavanaugh, 2020

Zachary S. C. Picker,
University of Sydney

Asteroid-mass range

$10^{-17}\ M_{sun} < M < 10^{-12}\ M_{sun}$

- Too large for usual Hawking radiation constraints
- Too small for microlensing
- Unconstrained?
  - Femtolensing: finite size, diffraction effects
  - Capture in stars
  - Ignition of white dwarfs

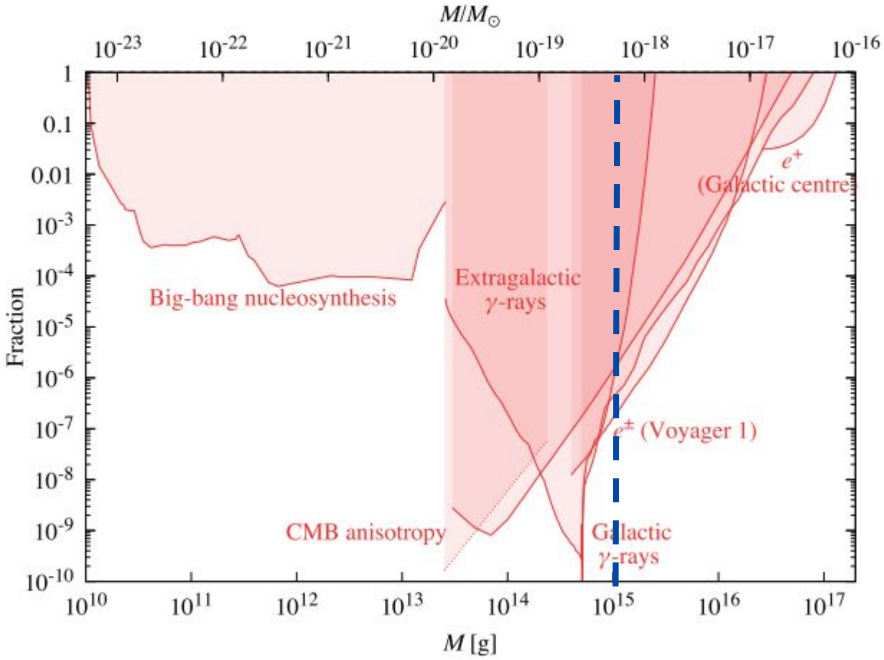
Monetero-Camacho et al, 2019

Zachary S. C. Picker,
University of Sydney

Evaporation-range constraints

$M < 10^{-16} M_{sun}$

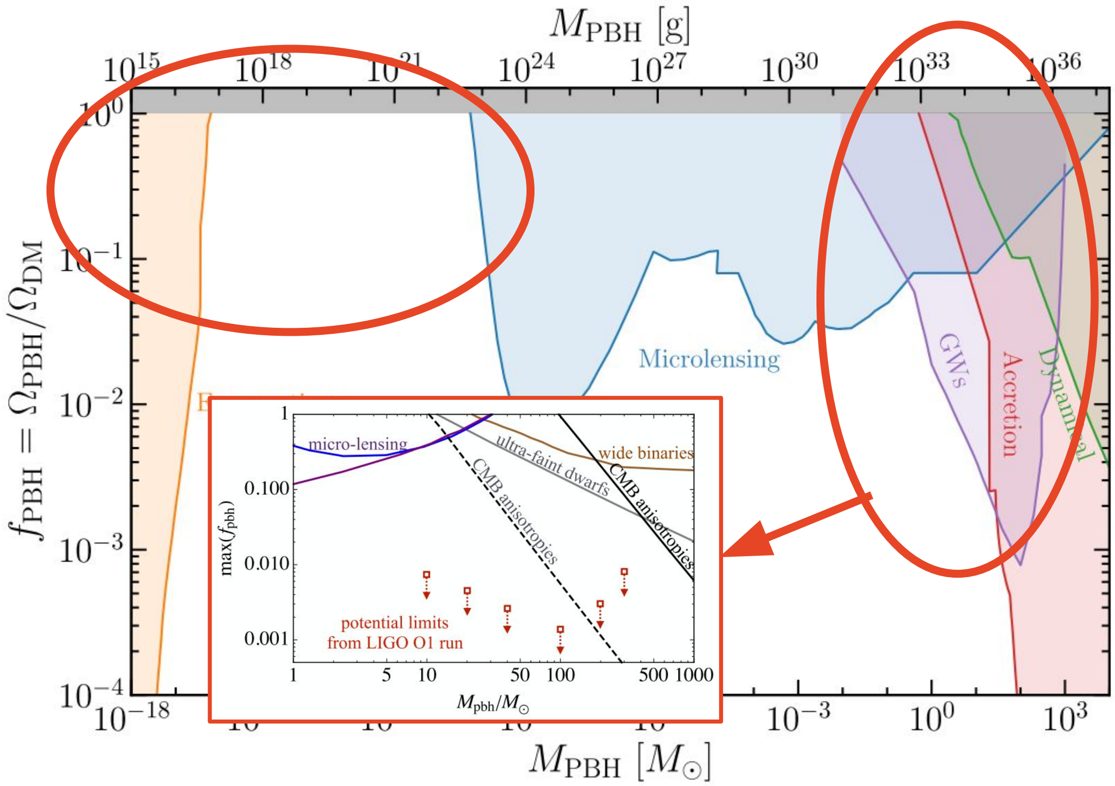
- PBH Lifetime
  - Stability constraint
- BBN
- CMB
- Gamma rays
- Galactic centre



Carr et al 2020

Zachary S. C. Picker,
University of Sydney

Areas of interest for cosmological PBHs



Green & Kavanaugh, 2020
Ali-Haïmoud et al, 2017

2. Cosmological PBHs

History

- Birth of PBH field is intimately connected to PBHs in cosmological backgrounds
- Carr & Hawking 1974: must use a BH solution which is asymptotically FLRW

THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel'dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4, pp. 758-760, July-August, 1966
Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.

BLACK HOLES IN THE EARLY UNIVERSE

B. J. Carr and S. W. Hawking

(Received 1974 February 25)

SUMMARY

The existence of galaxies today implies that the early Universe must have been inhomogeneous. Some regions might have got so compressed that they underwent gravitational collapse to produce black holes. Once formed, black holes in the early Universe would grow by accreting nearby matter. A first estimate suggests that they might grow at the same rate as the Universe during the radiation era and be of the order of 10^{15} to 10^{17} solar masses now. The observational evidence however is against the existence of such giant black holes. This motivates a more detailed study of the rate of accretion which shows that black holes will not in fact substantially increase their original mass by accretion. There could thus be primordial black holes around now with masses from 10^{-5} g upwards.

Zachary S. C. Picker,
University of Sydney

Cosmological PBH metrics

- Schwarzschild metric:
  - Embedded in flat, empty space
  - Mass is defined at infinity
- Cosmological solution:
  - Should be embedded in cosmological fluid
  - Asymptotically FLRW
  - *Local* mass definition
  - Still valid when cosmological horizon is close to BH horizon

Zachary S. C. Picker,
University of Sydney

Spherically symmetric metrics

$$\mathrm{d}s^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)\mathrm{d}\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- Can put any spherically symmetric metric in this form
  - (after suitable coordinate transformation)
- m_{MS} is known as the "Misner-Sharp" mass
  - Invariant measure of internal energy
  - Easy to see in these coordinates:

$$m_{\mathrm{MS}} = \int_V \mathrm{d}^3x\sqrt{-g}\,T^0_0$$

  - (quasi-) local and effective description of mass

Zachary S. C. Picker,
University of Sydney

Spherically symmetric metrics

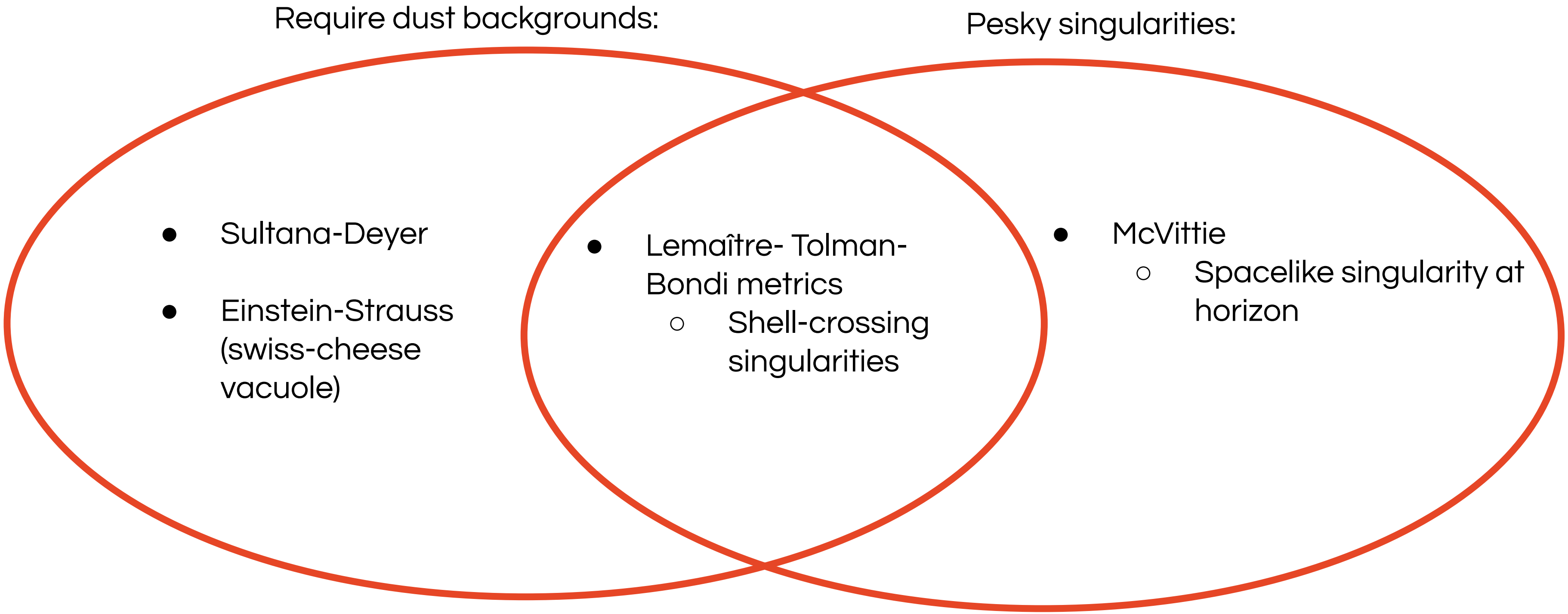
$$ds^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right) d\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

- Dynamical metrics don’t have a killing field
  - Instead we use the ‘Kodama vector’ to define timelike direction
  - Parallel to Killing field in the static limit
- Can then define a *geometrically preferred* time coordinate τ
  - In the above, the norm of the Kodama time translation vector is taken to coincide with the norm of the Kodama vector
  - Allows us to calculate surface gravity, four-acceleration with less ambiguity

Abreu & Visser, 2010

Zachary S. C. Picker,
University of Sydney

Brief survey of metrics



Generalized McVittie metrics ⇒ Thakurta metric

Zachary S. C. Picker,
University of Sydney

Thakurta metric

- Attractor solution of generalized McVittie metrics
- Simple & Elegant: $\mathrm{d}s^2 = a^2 \mathrm{d}s^2_{schw.}$
- $$\mathrm{d}s^2 = f(R)\left(1 - \frac{H^2R^2}{f^2(R)}\right)\mathrm{d}t^2 + \frac{2HR}{f(R)}\mathrm{d}t\,\mathrm{d}R - \frac{\mathrm{d}R^2}{f(R)} - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$
- $f(R) = 1 - 2Gma(t)/R$
- m is the 'physical' mass of the overdensity (or in static limit, today)
- Sourced by imperfect fluid with radial heat flow:

$$
\begin{aligned}
T_{\mu\nu} &= (\rho + P)\,u_\mu u_\nu + g_{\mu\nu}P + q_{(\mu}u_{\nu)} \\
q_\mu &= (0, q, 0, 0)\ , \\
u_\mu &= (u, 0, 0, 0)\ .
\end{aligned}
$$

Zachary S. C. Picker,
University of Sydney

Thakurta metric

- Would like the Thakurta in the general form,

$$ds^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right) d\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

- Use the transformation:

$$d\tau = dt + \frac{HR}{f(R)} \frac{dR}{1 - \frac{2Gm_{\mathrm{MS}}}{R}}$$

  - n.b. away from the horizon, Kodama time and cosmic time ~coincide

- Quasi-local Misner-Sharp mass: $$m_{\mathrm{MS}} = ma(t) + \frac{H^2 R^3}{2Gf(R)}$$

Zachary S. C. Picker,
University of Sydney

Thakurta metric

- Quasi-local Misner-Sharp mass: $m_{\rm MS} = ma(t) + \dfrac{H^2 R^3}{2Gf(R)}$
- Apparent horizon: $R_{\rm b} = \dfrac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \approx 2ma(1+2\delta)$
  - Gives us a small parameter: $1/8 > \delta := HGma$
- Look at the two extremes
  - Near the horizon: $m_{MS} \approx ma$
  - Far from the horizon: $m_{\rm MS} \approx \frac{H^2 R^3}{2G} = \frac{4\pi}{3} R^3 \rho$ ← *Total* energy density!

  $R \gg \left(2Gma/H^2\right)^{1/3}$

Zachary S. C. Picker,
University of Sydney

Thakurta black hole model

- Form from overdensity as usual: $m = \gamma m_H$
- Misner-Sharp mass is a *local* effective mass
  - Shouldn't use to calculate global parameters
  - Global BH energy density: $\rho_{\rm BH} = mn \propto a^{-3}$
  - n: PBH number density
- Can only use the Thakurta metric until BHs decouple
  - When their local dynamics is no longer dominated by cosmological fluid
  - eg. binary formation, virialization into galaxies
  - Once decoupled, they 'regain' an ~Schwarzschild description with mass m

Zachary S. C. Picker,
University of Sydney

3. Thakurta DM phenomenology

Binary abundance bounds

- Outline of process:
  - Black holes are randomly distributed at formation in the early universe
  - If BHs are sufficiently close, they can decouple from the Hubble flow to form a binary
  - We want to model the resulting distribution of binaries
    - Calculate the merger rate that LIGO could detect
- Assumptions:
  - Monochromatic mass spectrum
  - No clustering of PBHs (see: Clesse, Garcia Bellido 2017)

Zachary S. C. Picker,
University of Sydney

Binary formation

- Average BH separation today

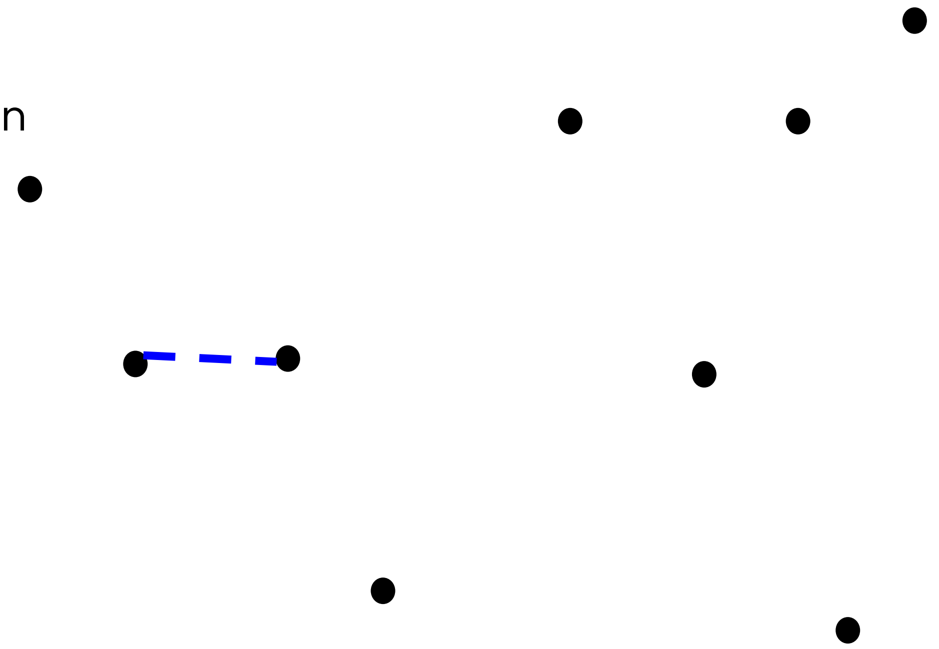
$$\bar{x}_0 = \left(\frac{m}{f_{\rm PBH}\rho_{\rm cr}\Omega_{\rm DM}} \right)^{1/3}$$

$$\approx \frac{1.2\ \rm kpc}{f_{\rm PBH}^{1/3}} \left(\frac{m}{30 M_\odot} \right)^{1/3}$$

- At earlier times,

$$\bar{x} = a(t)\ \bar{x}_0$$

Gives the physical average distance



Zachary S. C. Picker,
University of Sydney

Binary formation

- Semimajor axis:

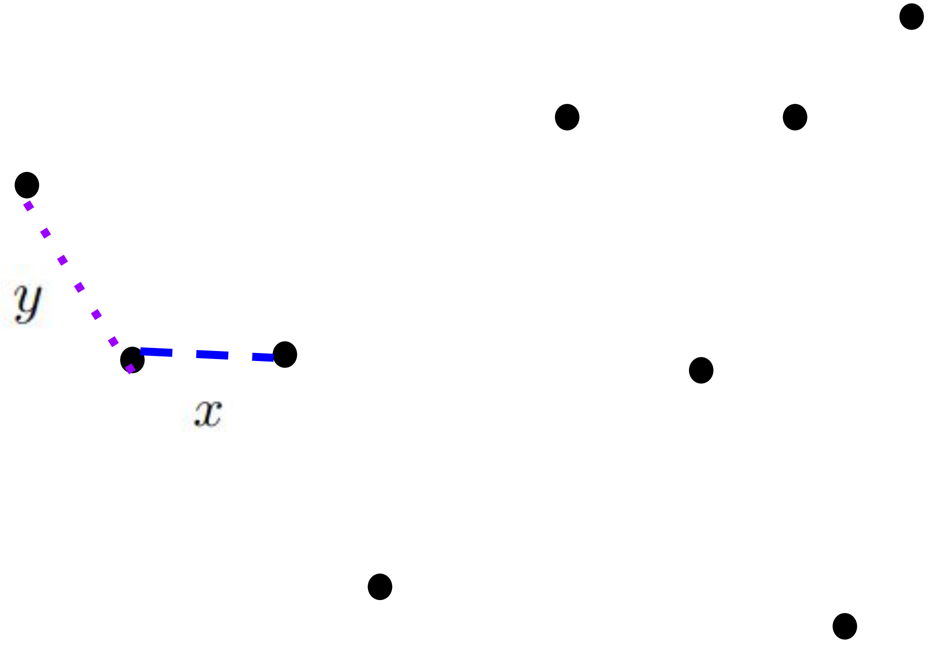
$$\mathfrak{a} = \alpha x$$

- Semiminor axis:

$$\mathfrak{b} = \beta \left(\frac{x}{y} \right)^3 \mathfrak{a}$$

- Eccentricity:

$$e = \sqrt{1 - \left(\frac{\mathfrak{b}}{\mathfrak{a}} \right)^2}$$



- Distribution function for BH separations in $(x, x + dx)$, $(y, y + dy)$:

$$dP = \frac{9}{\bar{x}^6} x^2 y^2 dx dy$$

Zachary S. C. Picker,
University of Sydney

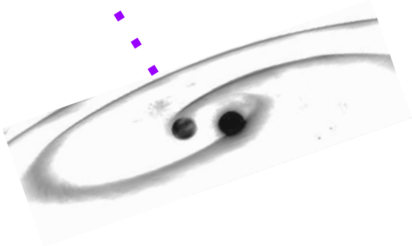
Binary formation

- If decoupling condition is satisfied at

$$z = z_{\rm dec}$$

- It coalescences with time

$$\tau_{\rm b} = \frac{3}{85}\frac{\mathfrak{a}_{\rm dec}^4(1-e_{\rm dec}^2)^{7/2}}{r_{\rm s}^3}$$



- Can rewrite the distribution function in terms of coalescence times and integrate over eccentricities

  - Event rate = $n_{\rm BH}\, dP\left(\tau_b \approx t_0\right)$

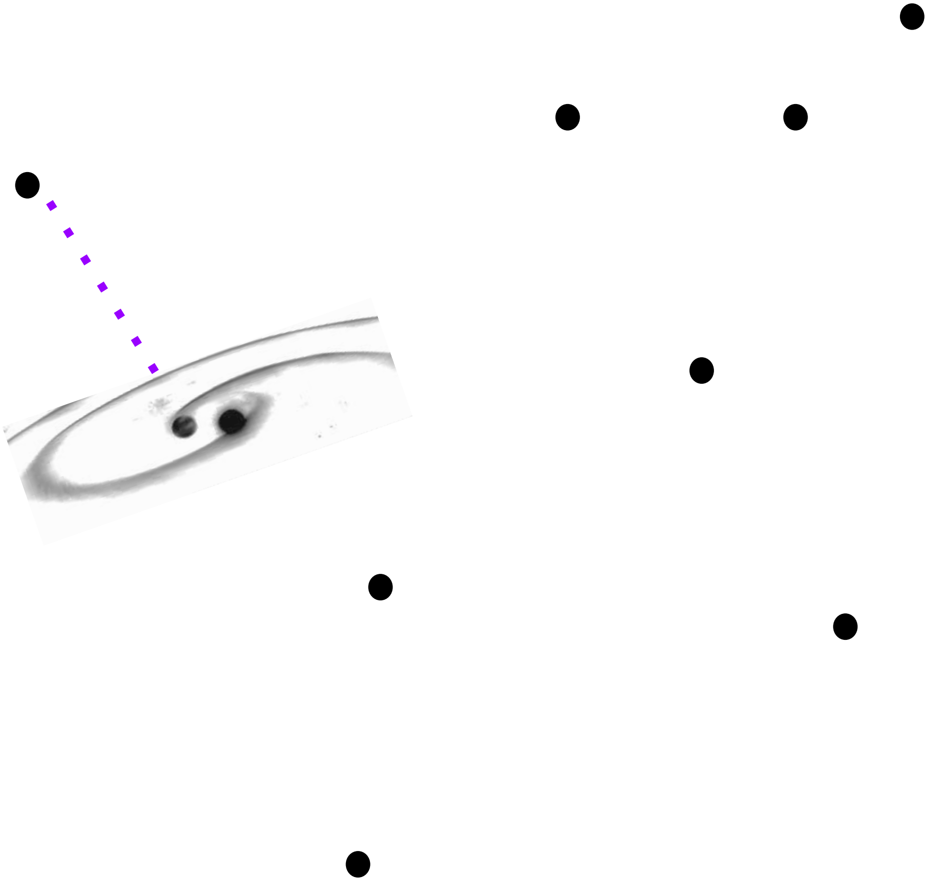
Zachary S. C. Picker,
University of Sydney

Decoupling condition

Schwarzschild decoupling:

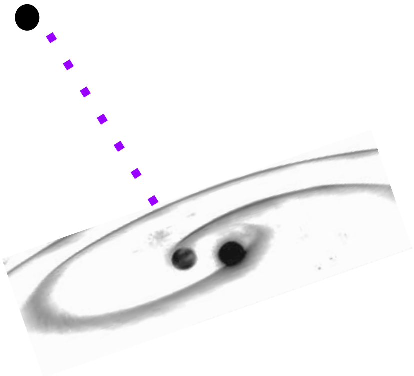
$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{Gm}{R^2}$$

$$\frac{m}{V} \gtrsim \rho$$



Zachary S. C. Picker,
University of Sydney

Decoupling condition in detail



Schwarzschild decoupling:

$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{Gm}{R^2}$$

$$\frac{m}{V} \gtrsim \rho$$

Thakurta Decoupling:

$$\ddot{R} = -\frac{Gma}{R^2} + \frac{\ddot{a}}{a}R$$

$$\frac{ma(t)}{V} \gtrsim \rho$$

$$E = -GM\mu a^2/(2R)$$

$$\dot{R}/R = -\dot{E}/E + 2H$$

$$\dot{E}/E > 2H$$

Zachary S. C. Picker,
University of Sydney

Thakurta BH quadrupole power

- For Keplerian motion: $d = \frac{\mathfrak{a}\left(1-e^2\right)}{1+e\cos\psi}$ $\quad \dot{\psi} = \frac{\sqrt{2Gm\mathfrak{a}\mathfrak{a}\left(1-e^2\right)}}{d^2}$
- The nonzero elements of the quadrupole tensor are:

$$Q_{xx} = \frac{1}{2}ma\ d^2\cos^2\psi \qquad Q_{yy} = \frac{1}{2}ma\ d^2\sin^2\psi \qquad Q_{xy} = Q_{yx} = \frac{1}{2}ma\ d^2\cos\psi\sin\psi$$

- The gravitational wave power is given by:

$$P = \frac{G}{5}\left(\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3}\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3} - \frac{1}{3}\frac{\mathrm{d}^3Q_{ii}}{\mathrm{d}t^3}\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3}\right)$$

- Power for Thakurta black holes:

$$P = -\frac{64}{5}\frac{G^4m^5a^5}{\mathfrak{a}^5\left(1-e^2\right)^{7/2}}\left[\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)+\ldots\right] \approx P_{\mathrm{Schw.}}a^5$$

Zachary S. C. Picker,
University of Sydney

Thakurta decoupling

- After decoupling, however, the PBHs follow the *usual* Schwarzschild coalescence time
- So we can combine the decoupling condition $\dot{E}/E > 2H$ with:

$$P = -\frac{64}{5}\frac{G^4 m^5 a^5}{\mathfrak{a}^5 (1-e^2)^{7/2}}\left[\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)+\ldots\right] \approx P_{\mathrm{Schw.}} a^5$$

$$\tau_{\mathrm{b}} = \frac{3}{85}\frac{\mathfrak{a}_{\mathrm{dec}}^4 (1-e_{\mathrm{dec}}^2)^{7/2}}{r_{\mathrm{s}}^3}$$

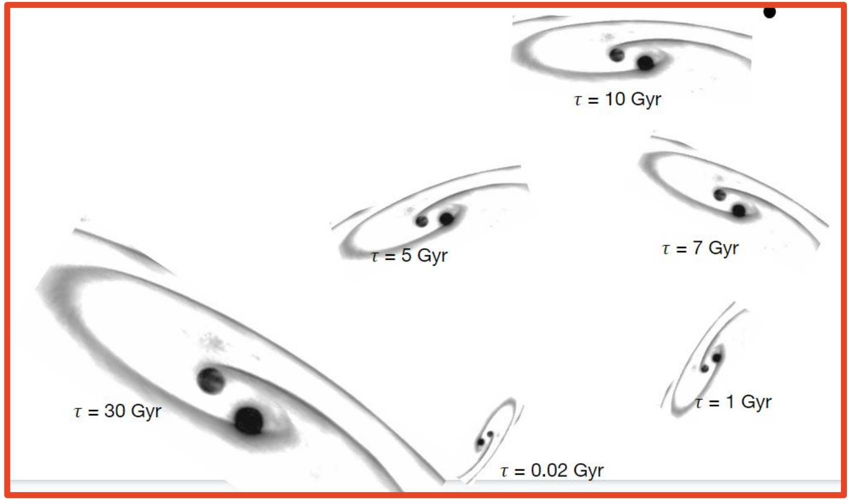
$$(1+z_{\mathrm{dec}})^3 H(z_{\mathrm{dec}}) < \frac{1}{\tau_b}\frac{96}{425}\left(1+\frac{73}{24}e_{\mathrm{dec}}^2+\frac{37}{96}e_{\mathrm{dec}}^4\right)$$

Zachary S. C. Picker,
University of Sydney

Binary abundances

Schwarzschild PBHs:

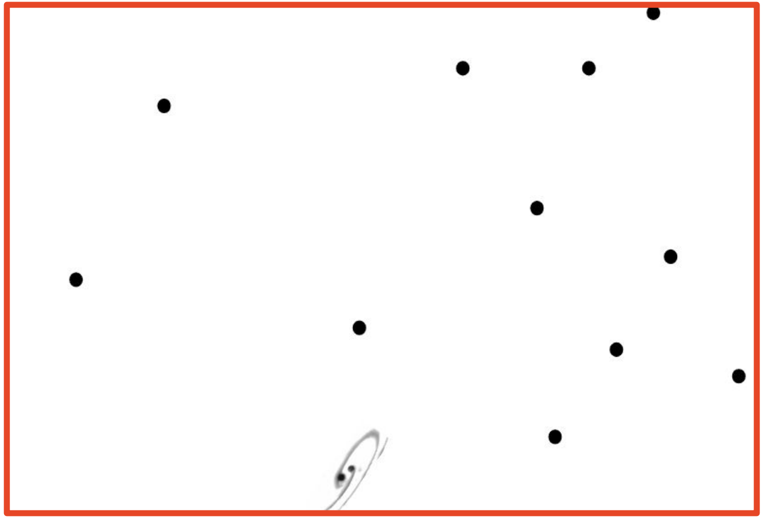
At matter-radiation equality:



Many of these coalesce ~today

Thakurta PBHs:

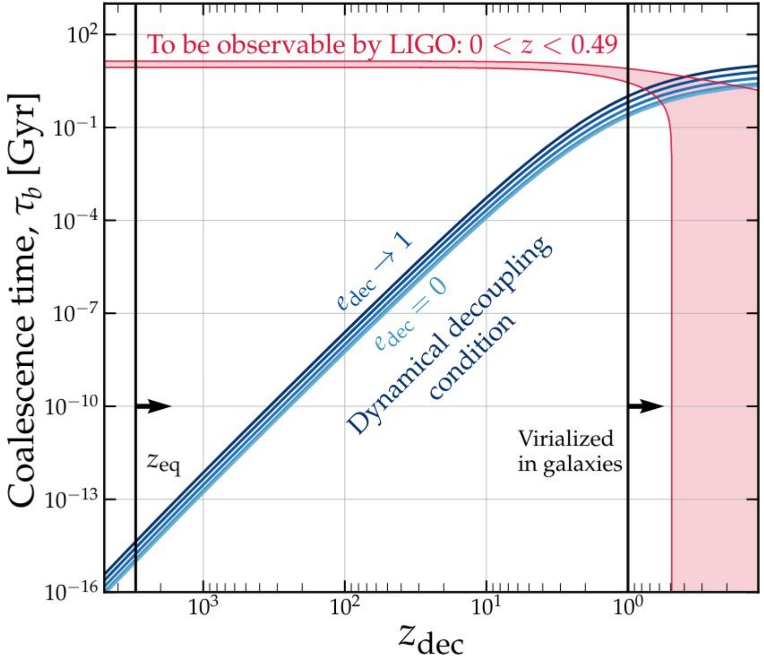
At matter-radiation equality:



τ_{max} ~ 100 sec (!)

Binary abundances

$$(1+z_{\rm dec})^3 H(z_{\rm dec}) < \frac{1}{\tau_b}\frac{96}{425}\left(1+\frac{73}{24}e_{\rm dec}^2+\frac{37}{96}e_{\rm dec}^4\right)$$



arxiv:2008.10743:

Eliminating the LIGO bounds on primordial black hole dark matter

Céline Bœhm,[a] Archil Kobakhidze,[a] Ciaran A. J. O'Hare,[a] Zachary S. C. Picker,[a] Mairi Sakellariadou[b]

Zachary S. C. Picker,
University of Sydney

Hawking evaporation

- Interested in smallest PBH which survives until today
  - ‘Critical mass’
  - Smaller PBHs cannot be a dark matter candidate
  - Why?
- ‘Stability constraint’:
  - You *could* have smaller PBHs today…
  - Extremely sensitive to PBH mass at formation
  - Cannot populate DM today
- Further constraints from effect of radiation on CMB, BBN, cosmic rays...

Zachary S. C. Picker,
University of Sydney

Thakurta surface gravity

- Black hole horizon: $$R_\mathrm{b} = \frac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \quad \approx 2ma(1+2\delta)$$

$$1/8 > \delta := HGma$$

- Surface gravity is relatively well-defined in our coordinate system:

$$\kappa = \frac{1 - 2\left(\frac{\partial}{\partial R} M_\mathrm{MS}(R_\mathrm{b})\right)}{2R_\mathrm{b}}$$

- For the Thakurta metric:

$$\kappa \approx \frac{1}{2Gma}(1 - 6\delta) \approx \frac{2\kappa_\mathrm{Schw.}}{a}$$

Abreu and Visser 2010

Zachary S. C. Picker,
University of Sydney

Thakurta Temperature

- We define temperature in the standard way: $T = \kappa/2\pi$
- Assume radiation follows the Stefan-Boltzman law: $\dot{U}_H = -\sigma T^4 A$
  - Measured by an observer far from BH and cosmological horizon
- T is explicitly time-dependent, so need to check thermality:

$$\left|\dot{T}\right| < \left|\dot{U}_H\right|$$

$$\downarrow$$

$$240\ Gma \lesssim 1/H$$

- We assume black holes form when $2Gm \sim 1/H$, so this is easily satisfied

Zachary S. C. Picker,
University of Sydney

Thermodynamic identity

- ‘Easy’ derivation of Hawking radiation using

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = T\frac{\mathrm{d}S}{\mathrm{d}\tau} - P\frac{\mathrm{d}V}{\mathrm{d}\tau} \qquad S = A/4$$

- Internal energy U changes as a result of radiation *and* from growing horizon:

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = -\sigma T^4 A + 2\delta$$

- Can solve the thermodynamic identity for the physical mass loss:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{1}{1920\pi G^2 m^2 a^2} = \frac{8}{a^2}\left(\frac{\mathrm{d}m}{\mathrm{d}t}\right)_{\mathrm{Schw}}$$

Zachary S. C. Picker,
University of Sydney

New critical mass

- Want to find critical mass m_* which evaporates by matter-radiation equality:

$$\int_0^{m_*} \mathrm{d}m\ m^2 \propto \int_{t_{eq}}^{t_f} \frac{\mathrm{d}t}{a(t)^2}$$

- The formation time (or redshift) is also a function of mass:

$$z_\mathrm{f}(m) = \left(\frac{2GmH_0\sqrt{\Omega_r}}{\gamma}\right)^{-1/2}$$

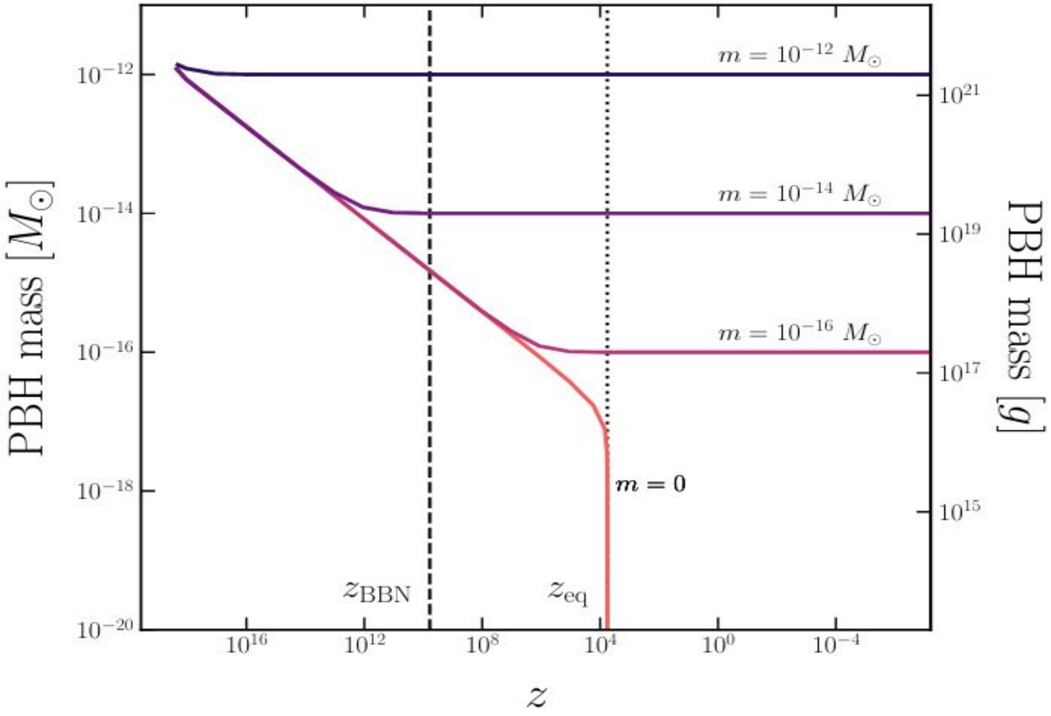
- Approximate analytic solution for critical mass:

$$m_* \sim 9.6 \times 10^{-13}\ M_\odot \left(\frac{\gamma}{0.2}\right)^{\frac{1}{7}} \left(\frac{h}{0.67}\right)^{-\frac{3}{7}} \left(\frac{\Omega_\mathrm{r}}{5.4\times 10^{-5}}\right)^{-\frac{3}{14}}$$

Zachary S. C. Picker,
University of Sydney

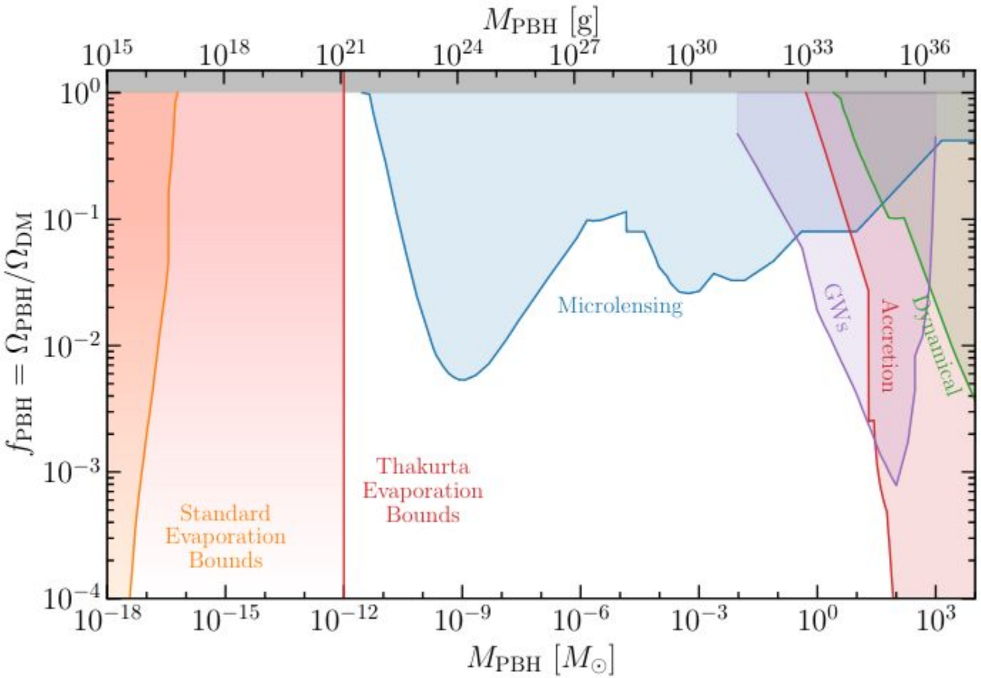
Hawking evaporation - critical mass

- Can also solve numerically
- 'Stability constraint' is readily visible here
    - Even an extended mass distribution will leave you with vast majority of BH masses today above m_* (or totally decayed)



Zachary S. C. Picker,
University of Sydney

Hawking evaporation - constraints



arxiv:2103.02815:

Navigating the asteroid field: New evaporation constraints for primordial black holes as dark matter

Zachary S. C. Picker

4. Conclusions

Zachary S. C. Picker,
University of Sydney

Aims?

1. Engage community

2. Thakurta phenomenology

Zachary S. C. Picker,
University of Sydney

Aims?

1. Engage community

2. Thakurta phenomenology

Reply to "Comment on: Cosmological black holes are not described by the Thakurta metric"

↓

Story about: discussion over "addendum to: questions regarding "flaws within the article: remaining tensions with "uncertainty around: regarding: "reply to: comment on...

↓

?

Zachary S. C. Picker,
University of Sydney

Aims?

1. Engage community

2. Thakurta phenomenology

- Avoid binary abundance bounds
  - Open up 30 M_{sun} window?
- Hawking evaporation
  - Close asteroid window?
- Lots more work to be done:
  - Accretion (soon! hopefully...)
  - More exotic processes...
  - Better Thakurta understanding
  - Better cosmological BH metrics?

Thanks for listening!